Stylized Facts

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Outline

According to Cont (2001) (also avalable here).

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> *statistical properties of financial time series, common across a wide range of instruments, markets and time periods.*

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We are studying (asset) returns $R_t = P_t/P_{t-1} - 1$.

No Autocorrelation

- Returns of longer periods do not show significant autocorreations
- For shorter periods (20 min) the microstructure effects take place (i.e. the bid-ask bounce causing a slight mean reversion)

Heavy tails

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- The α of Student distribution equals to its degree.

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- Mandelbrot [1963] shows empirically that the tails of price returns (of cotton) are fat
- Large literature proves the tail index of stock returns to be between two and five

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- There exit several methods of removing the bias (e.g. Huisman [2001])
- See e.g. Horák and Šmíd [2009] for details (may be downloaded here)
- The distribution of the Hill estimator h computed from k values is
$$N\left(\gamma, \gamma/\sqrt{k}\right) \doteq N\left(\gamma, h/\sqrt{k}\right)$$

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  Hint for the comparison: If both the sub-samples are of equal size end if the returns were independent, then, given

$$H_0 : \text{Both the indices are equal to the same value } \gamma$$

  it both h_- and h_+ would be assymptoticaly normal:

$$h_-, h_+ \dot{\sim} N(\beta + \gamma, \gamma^2/k)$$

  where β is the bias, and, moreover h_+ and h_+ would be independent. Using this, we would get

$$\frac{\sqrt{k}(h_+ - h_-)}{\sqrt{2}h_+} \doteq \frac{\sqrt{k}(h_+ - h_-)}{\sqrt{2}\gamma} \dot{\sim} N(0,1)$$

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- Returns with larger periods seem "more normally distributed".
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  To test this, we may compare the Hills estimators, however, they have to be based on samples of the same size!
- However, it seems true that the (type of the) distribution varies with the length of period

Volatility clustering

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- As a consequence of v.c., we may observer positive autocorrelation of the square returns - the decay of the autocorrelation function is very slow.
- Mathematically, v.c. may be modeled by GARCH models
- However, while the conditional returns in GARCH should be normal, it is found fat-tailed

Tails via Extreme Value Theory

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Tails via Extreme Value Theory

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- This suggests a “distribution free” method to estimate the tails
- In particular, we can split our sample into subsamples, get the extremal values and estimate the three parameters by the maximum likelihood.

Some More Stylized Facts

- Leverage effect: most measures of volatility of an asset are negatively correlated with the returns of that asset.

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- Volume/volatility correlation: trading volume is correlated with all measures of volatility.
- Long memory of trade signs (Wyart at al. 2008): If we count 1 for a buy and -1 for sell than the autocorrelation of the resulting series will decay slowly.

Homework

Each student should choose a different stylized fact and statistically verify it. Possible data sources: finance.yahoo.com, www.stocktrading.cz.

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